Construction and Consequences of Coloured Charges in QCD

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CONSTRUCTION AND CONSEQUENCES OF COLOURED CHARGES IN QCD[a]

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This talk reports on work aimed at improving our understanding of charged states in gauge theories. Emphasis is placed on different ways of implementing the gauge invariance of physical states. QED perturbative calculations are used to stress that gauge invariance is not enough: only a subset of such states have a direct physical significance. In QCD a non-perturbative obstruction means that quarks and gluons cannot form physical states and as such these particles are not truely observables. This sets a fundamental limit on the constituent quark model.

1 Charged States in QED

Gauss's law tells us that physical states in gauge theories have to be locally gauge invariant. This has many implications for any discussion of charged particles — such as electrons or quarks[1]. In QED the basic fermion is not gauge invariant but one way of making it so is to attach a 'string' to it:

$$\psi_\Gamma(x) := \exp\left(ie\int^x dz_\mu A^\mu(z)\right)\psi(x)\,, \tag{1}$$

where the line integral is over some path, Γ, from infinity to x. While this is gauge invariant if the gauge transformations fall off to constants at infinity, it has no physical signicance since the flux is compressed on to the path of the string. However, we may use it as a gauge invariant initial state and watch it evolve. If we assume that the charged particle is very heavy, i.e., static, then the Hamiltonian is just $\frac{1}{2}\int E_i^2 + B_i^2$, and the evolution is exactly solvable[2]. Essentially what happens is that the charge generates a Coulombic field and the string radiates away to infinity.

To see how to describe this final state we recall[3] that in QED

$$\psi_{\mathrm{f}}(x) := \exp\left(ie\int d^4z f_\mu(z,x)A^\mu(z)\right)\psi(x)\,, \tag{2}$$

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is gauge invariant if $\partial^z_\mu f^\mu(z,x) = \delta^{(4)}(z-x)$ holds. Clearly there are many possible solutions to Eq. 2 and indeed Eq. 1 may be understood in this way.

Another solution to Eq. 2 is given by $f_0 = 0$, $f_i = 4\pi\delta(z_0 - x_0)\partial^z_i 1/|z-x|$, with which we have

$$\psi_{\rm c}(x) := \exp\left(ie\frac{\partial_i A_i}{\nabla^2}(x)\right)\psi(x)\,. \tag{3}$$

This is so defined as to be local in time, which has obvious advantages, but it is spatially non-local. It may be seen that this is in fact a *necessary* consequence of Gauss's law for any description of a physical charged state. (An operator describing a charged state cannot commute with the charge operator, but this last can with the help of Gauss's law be rewritten as: $\int \rho \to \int \partial_i E_i$ and this in turn may be re-expressed as a surface integral over the sphere at infinity. A description of a charged particle must therefore include a non-local electromagnetic cloud, a dressing, around the charge. The non-covariance of Eq. 3 is justified in detail elsewhere[1].) Using the fundamental commutator, $[E_i(x), A_j(y)] = i\delta_{ij}\delta^{(3)}(x-y)$, one finds that the commutator of the electric field with the charged field of Eq. 3 yields the field times the change in the electric field we would expect of a static charge. This is what Eq. 1 actually evolves into in the study described above. The extension of Eq. 3 to charges with a constant velocity, i.e., a gauge invariant description whose commutators with E_i and B_i yield the electric and magnetic fields associated with a charge moving with velocity $v = (v^1, 0, 0)$ has been recently developed[1]:

$$\begin{aligned}\psi_v(x) = \exp\Bigg(&-\frac{e}{4\pi}\frac{1}{\sqrt{1-v^2}} \\ &\times\int d^3z\,\frac{(1-v^2)\partial_1 A_1(x^0,z) + \partial_2 A_2(x^0,z) + \partial_3 A_3(x^0,z) - vE_1(x^0,z)}{\left(\frac{(x_1-z_1)^2}{1-v^2} + (x_2-z_2)^2 + (x_3-x_3)^2\right)^{\frac{1}{2}}}\Bigg)\psi(x)\,.\end{aligned} \tag{4}$$

It should be noted that this is not just a Lorentz boost of Eq. 3 and further that a gauge invariant term (dependent on E_i) is present here. This last point further exemplifies the fact that gauge invariance is only a necessary requirement for a physical state and that, e.g., energy considerations must also be taken into account in any more realistic description.

To test the above interpretation of the dressed charges and to study the practicability of working with such variables various perturbative calculations have been carried out[1,4,5]. Recall that the mass-shell renormalisation of the usual fermion propagator in QED is, in general, marred by an infra-red divergence and that this divergence is understood as due to a neglect of the electromagnetic cloud surrounding the charge. The two-point function of the dressed charges was studied for the above-mentioned dressings which, it is again stressed, are supposed to correspond to charges moving with a particular velocity. This leads to the *prediction* that there will be no infra-red divergence if the propagator is taken on-shell at $p = m\gamma(1, v^1)$. This prediction has been *verified* by explicit calculation at one loop for an arbitrary velocity[4,5]. Since the infra-red divergence is due to the slow fall-off of the Coulombic interaction, the anomalous magnetic moment of the electron does not contribute. This means that the same dressing should lead to infra-red finite results for both fermionic and scalar QED. This has also been verified explicitly.

2 Charged States in QCD

A difference between QCD and QED is that the colour charge of the former theory

$$Q^a = \int d^3x \left(J_0^a(x) - f^{abc} E_i^b(x) A_i^c(x) \right) , \tag{5}$$

is not locally gauge invariant. It can, however, be demonstrated[1,6] that it has a gauge invariant action on gauge invariant states. (One may rewrite the colour charge with the help of Gauss's law and then express it as a surface integral; if local gauge transformations fall off at spatial infinity to unity, the action of the charge is gauge invariant.) It follows[1,6] that the colour statistics of coloured quarks dressed with coloured glue to form a gauge invariant whole will be just that of the naive quark model. So how do we dress the quarks? In perturbation theory one can extend the analysis of Eq. 2 to QCD order by order. Another way to extract the equivalent of Eq. 3 is to consider a quark field with a path ordered exponential attached to it — the QCD equivalent of Eq. 1. It is possible order by order in perturbation theory to factor out the dependence on the path of the string, Γ. This yields at order g^2 for a path fixed in one time slice:

$$\psi_\Gamma(x) := \mathcal{P} \exp\left(g \int^x dz_i A_i(z) \right) \psi(x) \to \mathcal{N}_\Gamma(x) \psi_{\mathrm{c}}^{g^2}(x) + O(g^3) , \tag{6}$$

where $\mathcal{N}_\Gamma(x)$ is path dependent but gauge invariant to this order in g and

$$\begin{aligned} \psi_{\mathrm{c}}^{g^2}(x) \;:=\; & \left(1 + g \frac{\partial_i A_i}{\nabla^2}(x) + \frac{g^2}{2} \left(\frac{\partial_i A_i}{\nabla}(x) \right)^2 \right. \\ & \left. - \frac{g^2}{2} f^{abc} T^a \frac{1}{\nabla^2} \left(A_j^b \frac{\partial_j \partial_i A_i^c}{\nabla^2} \right)(x) - \frac{g^2}{2} f^{abc} T^a \frac{\partial_j}{\nabla^2} \left(A_j^b \frac{\partial_i A_i^c}{\nabla^2} \right)(x) \right) \psi(x) \, . \end{aligned} \tag{7}$$

This is to order g^2 a gauge invariant, path independent description of a quark and the natural extension of Eq. 3.

It is useful to recall that in QCD we have, beyond lowest order in perturbation theory, no equivalent of the Coulombic electric field. This means that after finding gauge invariant dressings, it is not easy to understand what physical significance they have. Perturbative studies, energy minimisation etc. will all have their role to play. However, this talk will conclude by pointing out a fundamental obstruction to the construction of any gauge invariant dressed quark. In general a gauge invariant dressed quark may be written as

$$\psi_{\mathrm{h}}(x) = h^{-1}(x)\psi(x)\,, \text{ if under a gauge transformation } h \to h^U = Uh\,. \qquad (8)$$

This same field dependent h also defines a gauge invariant gluon. The question is now how do we find h? It may be shown[1] that the above transformation property of such an h means that not only may a gauge choice be used to construct an h, but any h may be used to construct a gauge fixing. We see now that Eq. 7 corresponds to Coulomb gauge. Since we know that good gauge fixings in QCD do not exist (the Gribov ambiguity) it is impossible to construct a physical quark state. This naturally explains why isolated quarks are not observed: such states cannot be constructed outside of perturbation theory. For more details and some suggestions as to how to obtain the scale at which the constituent quark model breaks down, we refer to Ref. 1.

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